# Extended Fermat's Last Theorem or Fortunado's Second Conjecture or Theorem and Solution to Beal Conjecture Using Gap Analysis or Difference Analysis 

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#### Abstract

Fermat's last theorem has been a discussion for a very long time. The author of this paper had come up with another conjecture which could be further evaluated, the extension of Fermat's or extended Fermat's. Will this be an easy one, equally hard or harder than the Fermat's. This paper also prove Beal's Conjecture using gap analysis or difference analysis.


Keywords: Difference Analysis or Gap Analysis, Fermat's Conjecture, Fermat's Last Theorem, Simple Solution, Table Analysis.

## 1. INTRODUCTION

In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture, especially in older texts) states that no three positive integers $a, b$, and $c$ satisfy the equation $a^{n}+b^{n}=c^{n}$ for any integer value of $n$ greater than 2 . The cases $n=$ 1 and $n=2$ have been known since antiquity to have infinitely many solutions. (Anonymous 2021)

Only one relevant proof by Fermat has survived, in which he uses the technique of infinite descent to show that the area of a right triangle with integer sides can never equal the square of an integer. His proof is equivalent to demonstrating that the equation

$$
x^{4}-y^{4}=z^{2}
$$

has no primitive solutions in integers (no pairwise coprime solutions). In turn, this proves Fermat's Last Theorem for the case $n=4$, since the equation $a^{4}+b^{4}=c^{4}$ can be written as $c^{4}-b^{4}=\left(a^{2}\right)^{2}$.
(Freeman 2005)
A proof was published by Andrew Wiles 1995.
A proof was published by Ismael Fortunado in 2021.

## Now an extension has been created,

No numbers $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{x}, \mathrm{y}$ and z aside from having the numbers 0,1 or 2 as a value(s) could satisfy $a^{x}+b^{y}=c^{z}$.
Beal's conjecture is a generalization of Fermat's Last Theorem. It states: If $A^{x}+B^{y}=C^{z}$, where $A, B, C, x, y$ and $z$ are positive integers and $\mathrm{x}, \mathrm{y}$ and z are all greater than 2 , then $\mathrm{A}, \mathrm{B}$ and C must have a common prime factor.
$A^{x}+B^{y}=C^{z}$

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## Solution for Beal's Conjecture

Look for counterexamples
For any integer,
$X^{3}+X^{4} \neq X^{5}$
Which is also equal to $X^{3} \neq X^{5}-X^{4}$
Let X be an integer.

A
Substitute 1 for X, .
$1 \neq 1-1$, difference of 1
Substitute 2 for X
$8 \neq 32-16$, difference of 8
Substitute 3 for X
$27 \neq 243-81$, difference of 135
As we compute, higher numbers will bring a higher difference. $(1,8,135, \ldots)$.
So X doesn't have counterexamples for $\mathrm{X}^{3}+\mathrm{X}^{4} \neq \mathrm{X}^{5}$

B
$X^{3}+X^{4} \neq X^{6}$
Substitute 1 for X , .
$1 \neq 1-1$, difference of 1
Substitute 2 for X
$8 \neq 64-16$, difference of 40
Substitute 3 for X
$27 \neq 729-81$, difference of 621
As we can compute, higher numbers will bring not zero difference or a higher difference.
Analysis from B has higher difference compared to A. $(1,8,35, \ldots)<(1,40,621, \ldots)$
As b and c increases, so the difference increases.
So 3 for a is not a counterexample for $\mathrm{X}^{\mathrm{a}}+\mathrm{X}^{\mathrm{b}} \neq \mathrm{X}^{\mathrm{c}}$

C
$X^{4}+X^{5} \neq X^{6}$
Substitute 1 for X ,
$1 \neq 1-1$, difference of 1
Substitute 2 for X
$16 \neq 64-32$, difference of 16
Substitute 3 for X
$81 \neq 729-243$, difference of 405
As we compute, higher numbers will bring a higher difference. ( $1,16,405, \ldots$ )
So 4 for a is not a counterexample for $X^{a}+X^{5=b} \neq X^{6=c}$
As we compute, as $b$ and $c$ increases, the difference increases.
So 4 for a is not a counterexample for $\mathrm{X}^{\mathrm{a}}+\mathrm{X}^{\mathrm{b}} \neq \mathrm{X}^{\mathrm{c}}$

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Note:
$\mathrm{Y}=\mathrm{X}+$ positive integer
$\mathrm{Z}=\mathrm{X}+\mathrm{a}$ higher positive integer
Since $\mathrm{Y}>\mathrm{X}$ and $\mathrm{Z}>\mathrm{X}$, the difference would be greater or not equal, negative.
Example using A
$X^{3}+X^{4} \neq X^{5}$
$1^{3} \neq 1^{5}-1^{4}<1^{3} \neq 3^{5}-2^{4}$, difference is greater $(1<243-16)$
$1^{3} \neq 1^{5}-1^{4} \neq 1^{3} \neq 2^{5}-3^{4}$, difference is not equal $(1 \neq 32-81$, negative $)$
$1^{3} \neq 1^{5}-1^{4}<1^{3} \neq 3^{4}-2^{5}$, difference is greater $(1<81-32)$
$1^{3} \neq 1^{5}-1^{4} \neq 1^{3} \neq 2^{4}-3^{5}$, difference is not equal $(1 \neq 16-243$, negative $)$

Repeat the analysis...5, $6,7, \ldots$ are not counterexamples also for a.
Since there is no counterexamples for a, there would be no counterexamples for the conjecture.

## 2. CONCLUSION

Fermat's last theorem could be extended this way. This proves also Beal's conjecture as a major subset. Fermat's last theorem is solved by the same author using Gap analysis or Difference Analysis. Beal's conjecture was solved using Gap Analysis or Difference Analysis. There will be no counterexample likewise. Since there is no counterexample, Beal's conjecture is correct and proven. The common prime factor is only an extension or a subset.

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